A STUDY OF METHODS FOR

NUMERICAL CONTROL OF MACHINE-TOOLS

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SECTION I

INTRODUCTION

The automatic control of machine-tools is not something really new. An early example is the punched-card-controlled loom made by Jacquard in 1804 for weaving figured fabrics. The development of automatically controlled machine-tools took place all through the 19th century, but the use of digital electronic computers opened a new field to machine control by extending the economical use of machine-tools to small series production by eliminating the human factor as a cause of error and by increasing the overall accuracy in production.

The objective of this thesis is to study and compare methods for numerical control of machine-tools. The problem concerns mainly the digital-to-analogue transformation by means of which design information is transformed to control information. The blue-print gives an analogue representation of the piece to be cut, and fundamental digital data give the precision. The final shape of the piece is determined by the path followed. The worker usually locates the cutter by hand while performing a digital-to-analogue transformation from the digital representation of the piece on the drawing into the analogue form of the piece itself.

The same transformation can be performed by other means, such as a machine-control system, and this latter type requires some type of data storage.

The best known and most widely used kind of storage is the master cam, which is an analogue storage. Recording continuous signals on magnetic tape is also an analogue storage; however, it is not the purpose here to study this kind of storage. Another possibility is the digital storage, by means of punched cards or tape. This kind of input may be of incremental type and it may also give the co-ordinates of certain points, certain locations of the tool and other indications such as feed-speed, spindle-speed and tool-turret rotation.

The main concern in the present study is the digital-to-analogue transformation under its various forms according to its applications, viz:

To position the work piece (or the tool) at some discrete locations with the highest accuracy, as for drilling, spot-melting etc...

To control the respective positions of work piece and tools at all times as for contour milling.

The principal conversion systems usually employed may be grouped arbitrarily into four sections:

(1) Those using stepping devices to produce incremental motions.

(2) Those using quantizers to measure motions.

(3) Those using digitizers to measure motions.

(4) Those using matrix decoding to generate analogue command signals.

As most of these systems give essentially discrete signals defining a finite number of points on the surface to be cut, it is of primary importance to interpolate the motion of the tool between these points when one wants to control the respective positions of work piece and tools at all times, in order to have a smooth surface. It is possible to imagine a system giving a number of controlled points large enough to be more accurate than the machine-tool itself, but this would be a waste in tape-preparation time and complexity of the control system.

As a conclusion, a short summary of data preparation will be followed by a short survey to show the industrial and economical consequences of numerically-controlled machine-tools, concerning the quality of production and its cost.

SECTION II

DIGITAL TO ANALOGUE TRANSFORMATION

Stepping Type Devices

When the input is of incremental type, a stepping type device may be used to perform the digital-to-analogue conversion. Each discrete signal causes an incremental motion of the device which may be either geared directly to the machine member itself or coupled to it through a servo mechanism. The stepping device may be a rocker, a stepping motor or a synchronous motor.

Example 1. -- Milling machine developed by Dr. Herman Cousins (1). This system does not aim at operating the milling-machine at the maximum of its possibilities. It has been devised so that it should be possible to produce parts too intricate to be produced on a machine-tool run according to ordinary methods.

Handling four co-ordinates simultaneously, the equipment uses a four channel tape on which one word takes two rows. For each step, four holes are punched on the tape, in one row or the other. This gives sixteen possible hole configurations, that is sixteen different values for the given step of the given co-ordinate. A small rocker brought towards the tape rocks one way or another, depending which of the two holes is punched. Then, when moving further, the rocker encounters a dog which forces it on in the direction in which it is moving. This last move closes electrical switches to provide any one of the sixteen different sets of voltages which are fed directly to a synchro motor.

The elementary step is 22.5 degrees of synchro rotation, corresponding to an incremental motion of 0.001 inch of the machine member. For such an angle, the synchro output torque is able to drive the cross-slide alone. A larger driving power would require a servo motor to aid the synchro motor in driving the load. Smaller elementary steps would apply for the same requirements, because they imply smaller synchro rotations, andthe couple developed by the synchro decreases rapidly when the angle of rotation decreases.

Example 2. -- Electric stepping motor (2). Some progress has been made in the past few years in this field. Roughly the stepping motor is built as follows: The stator is divided into n spaced sections or phases with the same number of poles in each phase, and the poles in the n phases are aligned axially, with this peculiarity, that the space between two consecutive poles in the same phase is equal to the width of the pole itself. The winding is made so that two consecutive poles are of opposite polarity. Three phase motors have been built with 54, 108 and even 360 steps per revolution, that is 18, 36 and 120 poles in each phase.

The rotor has also n phases with the same number of poles as the stator has, but each phase is one nth of the pitch out of phase with respect to the preceding one, all in the same sense (Fig.1-A). Therefore, when the teeth of one phase of the rotor are megnetically strapped into register with the associated stator teeth, the rotor teeth of the next phase to be energized project over the stator gap by two nths of a tooth width (Fig.1-B). Energizing the phases in this order, according to control pulses, will cause the motor to turn in discrete steps with a speed proportional to the rate of pulses. Reversal can be obtained by energizing the phases in opposite order.

A defect of this system is that the inertia of the rotor increases rapidly with the number of steps per revolution. This condition may become still worse if the rotor is wound in order to increase the output torque. Overshoot and oscillations around the equilibrium may be encountered. On the other hand, inertia is an interpolating factor between two consecutive points.

To suppress oscillations, electrical or electro-mechanical timing means have been used, but they are complicated and not very successful. Another system is a reversible over-running clutch, or brake (Fig.2). This is not an ideal device for although it prevents the rotor from swinging about its equilibrium position, it does not prevent the overshoot, which introduces in the stop position an error of a given percentage of the elementary step increment.

A three phase stepping motor driven from a punched tape can be used as follows (Fig.1-C):

The tape is read by a contact type device. A hole in the tape gives a pulse which is used to fire one of the thyratrons. The two other thyratrons are necessarily stopped through the use of the capacitors, "C". Each thyratron, when producing direct current, energizes one of the three phases of the stator, fastening the rotor in a given position. Firing the thyratrons in sequence makes the rotor run in a given direction. Rotation in the opposite direction is obtained by reversing the sequence of firing the thyratrons. The one-way brake must also be reversed with the help of the solenoid "C" and the antagonist spring "D" (Fig.2).

A basic step providing an increment of 0.001 or 0.002 inches can be used for a contour milling machine, inertia of the rotor providing an approximate interpolation between two points. Many other applications require only an accurate selection of discrete points without regard to the path followed between these points, such as drilling, punching and spot welding. However, in most of their applications, stepping devices are limited to the control of slowly varying motions rather than to end-point control where rapid shifts must be made from one place to another.

Counting Techniques Using Quantizers

When stepping devices are not directly suitable because of speed torque or dynamic limitation, closed-loop principles may be applied. Signals generated by a quantizer coupled to the moving members are compared with the input signals. As shown in Fig.3, for example, a reversible binary counter may be used to count continuously the difference between the number of input increments and the number of feedback increments. This method is used in the Ferranti (4) and Bendix (3) systems, as explained below. The net count standing in the counter at any time represents the position-error in the servo loop. This is converted to a proportional voltage which is then amplified to drive a servo motor.

This system can handle practically unlimited input rates, with any type of motor having adequate dynamic performance and power capability to drive the load. If the possible counting rates are sufficiently high and the quantized increments sufficiently small, the deterioration in servo performance resulting from the discrete nature of the feed-back and error sensing device in the loop is quite small. The count-to-voltage conversion which is required takes place within the servo loop in a place where accuracy is not important.

The principal disadvantage of systems using quantizers is that accurate knowledge of position depends on having correctly counted all increments from the last reference point; however, in the case of control systems which naturally produce incremental data, additional means to avoid difficulty on this account can frequently be justified by the relative simplicity of quantizers and their associated circuitry, as compared with absolute digitizers.

Considerable emphasis is placed on reliability and long life in a relatively unfavorable environment from the standpoint of dirt, oil and vibrations. As a result, sliding electrical contacts, as in slip rings and commutators, are usually avoided when at all feasible. Also, high resolution is desirable in rotating devices, in order to minimize the gearing required for coupling to a lead screw or rack-and-pinion drive.

Example 1. -- The numerically controlled cam milling machine developed by the Bendix Aviation Corporation (3. Fig. 5 and 14).

The machine is designed to produce three dimensional master-cams and engineering prototypes directly from numerical control information recorded on punched tape.

Machine configuration.

The machine resembles a small lathe, except that a high speed ball-end milling cutter is substituted for the tool. The work piece is rotated about its axis as the cutter moves axially and radially with respect to it. As indicated in Fig.5, the cutter is mounted on a cross-slide which moves perpendicularly to the spindle axis. The cross-slide, in turn, is mounted on a longitudinal carriage, providing motion parallel to the axis.

The work spindle is driven by an hydraulic motor, which also rotates a lead screw to drive the longitudinal carriage. As a result of the fixed gearing, the tool tends to trace a helical path on the surface of the work piece. The cross-slide is driven by a separate motor and screw in accordance with control signals synchronized to the spindle. The radius of the helix thus varies as the part rotates, to produce the required surface. A portion of the cutter center helix, "H", is sketched on the cam scheme shown inFig.4, with an exaggerated pitch for the sake of illustration.

Control system and interpolation

By virtue of the fixed gearing between the work-spindle and the longitudinal carriage, the problem of generating the required three-dimensional cam surface is reduced to a two dimensional one, that of controlling radius "r" of the helix "H" as a function of helix angle "A". Information for this purpose is provided from punched tape in form of binary-coded data. It consists principally of the difference in radius "Dr" betwen successive points along the helix, and the size of the angular interval "DA" which also implicitly specifies the spindle speed.

The primary function of the machine control system is to guide the tool when it is cutting between known points, along an arc of a spiral of Archimedes (cf.Digital interpolation, Section III). Because of this built-in interpolation feature, the machine requires only a relatively small amount of input data, compared to that which would be needed to generate the surface by specifying a sufficient number of separate cutter positions. Of course, the reference points must be chosen in such a way as to avoid sharp angles on the surface of the work, using, if necessary, smaller increments DA of the spindle angle of rotation.

Control information specifying the amount by which the cutter position is to be changed during each angular interval is read from the tape into the interpolating circuitry. Also supplied to the interpolator is information locating the angular position of the spindle. This spindle information originates from a pulse generator connected by precision gearing to the spindle drive motor and producing 256 ($=2^8$) electrical pulses for each five degrees of cam rotation, i.e. approximately fifty pulses per degree.

As spindle pulses are produced, the interpolator selects certain ones to be used in cross-slide command pulses. Each such pulse represents a desired cross-slide motion, and consequently cutter motion, of 0.0002 inch. If, for example, a rise of 0.05 inch is required in five degrees, 250 pulses will be selected from the 256 spindle pulses produced as the cam blank rotates through the five degree interval (0.0001 inch = 2.54 micron).

The interpolator consists of a binary operational multiplier (Fig.13), and two flip-flop storage registers, designed as "Temporary" and "Active" storage registers (Fig.14). As cutting proceeds through each angular interval, under control of the active storage register, information for the following interval is read into the temporary storage register. When the boundary between intervals is crossed, the new information is transferred to the active storage register and the tape begins to read information for the following interval.

This use of two registers, with practically instantaneous transfer from one to the other, allows continuous operation of the machine from one interval to the next. The multiplier is controlled by the difference information in active storage. It consists of a binary counter, fed from the spindle quantizer with appropriate gating to channel certain of the "non-carry" pulses of the command pulse line as explained in the Section III.

Cross-slide servo

The cross-slide is positioned by a high performance servo mechanism, in response to command pulses generated by the interpolator. Feedback is provided by a quantizer similar to that associated with the spindle, producing a pulse for each 0.0002 inch of slide motion. Such a pulse represents the real motion in the same terms that a command pulse represents desired motion. A reversible binary counter performs the error sensing function in the servo loop, by counting continuously both command and feedback pulses. Therefore the counter holds at any time the difference between the nunber of command pulses generated by the interpolator and the number of feedback pulses received from the cross-slide. This net count is converted into a proportional voltage by a resistor-rectifier decoding matrix and furnished as error voltage to the servo amplifier which operates a specially designed valve which meters fluid to the piston-type rotary hydraulic motor driving the cross-slide.

Careful attention was given to inertia, structural rigidity, friction and backlash in the machine design. In a personal interview with the writer, Dr. E. C. Johnson pointed out the fact that the big problem was in the exact construction of the hydraulic components, with appropriate dynamic compensation of the servo loop, to permit the realization of an extremely fast-acting cross-slide servo. Its velocity constant* is about 1500 per second and bandwidth well over 100 cycles per

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* The velocity constant is by definition the inverse of the derivative of the error function "E" in the closed-loop system:

$$\frac{I}{Kv} = \frac{d}{ds}\,\frac{E(s)}{R(s)}$$

The larger is Kv, the smaller will be the error variation for a given variation of the input "R".

The bandwidth ranges from zero frequency to the frequency where the second. While cutting a typical countour under normal operating conditions the average dynamic error ranges from 0.5 to 1.5 times the 0.0002 inch increment.

Example 2. -- For extremely accurate control of linear motions, a linear quantizer mounted directly on the machine member is desirable. Undue requirements on the accuracy and rigidity of lead screws and gearings are thereby avoided.

One type of high resolution linear quantizers utilizes an accurately ruled optical grating the same length as the maximum stroke to be measured (Fig.6). The Ferranti Research Laboratories (4) uses this device on a cam milling machine to measure the longitudinal and transversal motions of the tool. The grating has 5000 lines per inch. Another short grating of the same pitch serves as a cursor, but is slightly cocked, so that when illuminated from behind, light and dark bands will appear in a <u>moire</u> pattern, perpendicular to the relative motion of the gratings.

The observation of these bands with a photo-electric cell gives electrical pulses. The number of pulses shows the amplitude of the motion. The rate of the pulses shows the speed of the motion. The direction of the motion can be determined from the observation of the dark and light waves with a second photo cell out-of-phase with respect to the first one. If the phase angle is 180 degrees, 5000 lines per inch give 0.0001 inch per pulse.

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output gain is larger or equal half the power gain at zero frequency (here, from zero to more than one hundred c.p.s.). It is a measure of the speed of response.

This accuracy is available on the work piece only if the same accuracy can be expected from the machine tool. Not only the parts of the machine must be perfectly adjusted and fitted to each other, but the gratings must be well protected against shocks, bendings, oil and dust. The engraving of such gratings is also a very hard problem, for if one considers only the variation of length of the material due to variations of temperature, the accuracy reached by such devices may soon find a limit which could be passed only by measuring the piece itself, and not the motion of the cutter.

Coded Digitizers Systems

The position of the tool (or of the cross-slide) is measured by a number read on a mechanical or an electrical device. For instance, the device of Fig.7a (5), consists of discs divided into ten sectors, geared to each other with a turn ratio of one-tenth. There is the same number of significant digits as discs. The control unit can "read" the number by means of electrical contacts, with brushes, as shown on the figure.

The major defect in this case, is that the brushes may straddle more than two bars, if they are too large, or lie between two bars if they are too small. The use of star wheels can eliminate this difficulty.

Another example is shown in Fig.7b(5). The angular output, similar to the preceding one, is optically read under the form of a binary code, with the help of a set of photo-cells. The disc is divided in a certain number of either opaque or transparent sections, as shown on Fig. 7b. The ambiguity due to the possible overlapping of two adjacent sectors is still possible, though it is now possible to say that the error will only be with the adjacent number to be read, and a device can be made to compensate for this. Of course, the narrower the slit "F", the better is the accuracy in the reading.

The numbers read on an optical digitizer are directly compared with the data numbers read from the tape. The moving part to be controlled has reached the desired position when the circuits energized by the coded disc digitizer of the output measurement correspond to the circuits of the input data energized by the tape. As long as that correspondance is not realized for the most significant digits, the driving motor of the moving part (cross-slide, for instance) is run at full speed in the direction specified by the tape. If that last specification has been forgotten, the motion may start in the wrong direction and the cross-slide would reach the end of its run and come back in the right direction, after the action of the reversing switch at the end of the run, reaching the exact position without error, but with a waste of time.

To avoid the trouble of a direction-of-motion signal on the tape for short motions, one can use a sensing device giving automatically the proper sense of motion when for instance the two least significant digits of the output only are different from those of the input. An interesting example is the Hillyer drilling machine (6).

The Hillyer system uses an electromechanical drive and a reference scale operating from electrical contacts. The measuring method is an optically produced contact scale. No further details are given about the measuring and comparing devices, but they may be similar to those of the last example above.

Two motors position the tool and the work piece in two directions at right angles, thus providing complete two-dimensional freedom. The co-ordinates in these two directions refer to the same zero reference line, elimating all point-to-point measurement whose results are disastrous in case of cumulating error.

The machine has no directional sense until it has come to the immediate vicinity of a location point, and a special instruction for the initial direction to travel must be provided from the tape, as explained above, to suppress the possibility of useless back-and-forth travel. The tape specifies the X and Y co-ordinates in inches and thousandths, and actuates the same circuits as those controlled by manual setting. However, in this case, the most interesting hole-to-hole sequence is that which gives the shortest way, and not necessarily that following an X or a Y axis, as it would be in manual setting, for the machine can handle X and Y simultaneously without any more difficulty than X or Y alone. The travel time is the important factore here.

Matrix Decoding

The last category of conversion systems develops a measuring device which gives an analogue representation of the actual dimension. The problem is then to compare this analogue representation to the s tatic numerical representation as given from the data tape or punched cards. The tape acts as would a key board and, for instance, actuates a precision voltage divider to produce a voltage proportional to the desired dimensions. If the analogue representation of the actual dimension is a voltage, the feed drive is actuated until the two voltages are balanced.

An example of matrix decoding is seen in the Excello boring machine, as altered by the Minneapolis Honeywell Regulator Company, to be automatically controlled by tape (7). The rotating fixture is positioned by an electric motor, and the cross-slide by an hydraulic cylinder. The motions of the cross-slide and of the rotating fixture act on a potentiometer each, modifying the reference voltage to give a feedback voltage and indicate the linear and rotary positions. The minimum voltage variation corresponds to an increment of 0.0001 inch, over 8 inches linear range of the cross-slide, and 0.01 degree over 360 degrees range of the rotary fixture.

The reference voltages, generated according to the hole configuration on the tape, are compared to feedback voltages and the corresponding machine members are kept in motion until the voltages are balanced. Then the basic cycle of the machine runs until the hole is bored. Then the positioning-and-boring cycle is repeated until the program is finished.

Another similar example is the cam milling machine developed by Electric and Musical Industries Ltd. (8; see Fig.8). The tape gives the co-ordinates of a succession of points on the surface of the cam (radius in terms of angle). The holes in the tape allow the fingers of the reading machine to operate electrical switches as is common use in reading devices.

The table proceeds by steps if the reader uses only one set of rows at a time, corresponding to one point. If a set of relays, r_1 and r_2, takes in account the co-ordinates of two consecutive points, it is possible to set up proper voltages at the extremities of the reference potentiometer "p". The motion of the table is obtained from a series of servo mechanisms (A, B and C), controlled by the lead screw "J". The potentiometer "p", actuated by the same lead screw, gives a voltage "V" which depends on the position of the table. The comparison between V and the instantaneous voltage read on "p" gives an error signal which is later amplified, fed into the motor "L" to change the position of the lead screw "J" and to bring the table in the desired position.

Actually, the point "m" has not a fixed position on the potentiometer "p" but swings from one end to the other, in order to have a linear interpolation between two consecutive points of the surface. The study of this example will be resumed in the section "Interpolation", for the system has been improved in order to have parabolic interpolation using three consecutive points (cf.Section III).

To avoid the voltage dividers, a device using a pair of command and feedback synchros for each decimal digit provides more accuracy than what can be obtained from a single analogue element.

A punching unit developed by the General Electric Company (9), has four pairs of selsyns to take care of each of the two motions (Fig.9). The position data are given from punched cards by a numerical digits reading unit. The mechanical movement of these reading assemblies are combined to position simultaneously the rotors of each selsyn in a position exactly the same as if these rotors were geared together by successive gearings of ratio one tenth and were rotated to the total decimal angular position corresponding to that specified by the information of the cards.

The actual position of the slide is indicated by four selsyns which are actually geared to each other by gearings of ratio one-tenth. To selsyns only are used to indicate the turret angular position. For one inch of the cross-slide motion, the selsyns rotate respectively one, 10, 100 and 1,000 revolutions, and for one revolution of the turret, the corresponding selsyns rotate one and ten revolutions. A higher accuracy, as often necessary in military equipment, could be obtained with a higher number of selsyns.

In order to set up the two variable dimensions to their proper values, a discriminator circuit compares the two voltages in each pair of selsyns and their polarities, to produce direct current output voltages. These direct current voltages are applied, with their signs, to electronic reversing motor controls to operate the corresponding motor drive in the right direction to bring the system in the desired position where exact correspondence of voltages is realized.

The "error" voltage above is constant for large deviations, and proportional to the small deviations, as a consequence of the limited possibilities of servo mechanisms and to avoid overshoot. The correspondence of the voltages is realized by steps: The selsyn representing the most significant digit represents the hole number, but with a poor accuracy. The switching network (Fig.9-D.C) acts in order to realize a near zero coincidence for the data and feedback voltages corresponding to this digit, with the maximum power delivered to the driving motor. Then the control is switched to the second most significant digit and the same sequence of operation is repeated from digit to digit, that is from one pair of selsyns to another pair of selsyns until a complete coincidence is obtained. For the least significant digits, corresponding to small motions of the slide (or of the turret) the power delivered to the driving motor is proportional to the error, as explained above, to avoid overshoot and damaging the work piece or the tool.

SECTION III

INTERPOLATION

The purpose of interpolation is to generate a smooth surface between a limited number of discrete points which are accurately defined from the data. Usually the contour is either bi-dimensional or tri-dimensional, or, though tri-dimensional may be considered as bi-dimensional when one motion of the machine may be considered as a function of the two others. If truly tri-dimensional contour is to be cut, interpolation is obtained in two steps, by successive application of interpolating functions of one variable.

These interpolating functions may take various forms, the most used of which has been so far of polygonal type:

$$y = f(x) = a_o + a_1 x + a_2 x^2 + \text{---} + a_n x^n \qquad (1)$$

The coefficients a_k are usually determined by requiring that the curve of equation (1) pass through $n + 1$ given points if the degree of $f(x)$ is chosen equal to n. It may also be required that the slope of the curve vary continuously around a set of successive points, but little advantage seems to result from such possibilities.

The concept that one machine motion is a function of another one is satisfactory when one of the motions can be assumed to vary regularly in the same direction with limited rates of change of the other motion. When milling an irregular closed contour in an (x,y) plane, it may be useful to consider y as a function of x on some parts of the contour, and

and x as a function of y on the other part of the contour. Shifting of the independent co-ordinate may avoid the complication of double valued functions like

$$F\ (xy) = 0 \qquad (2)$$

which is more general.

Linear Analogue Interpolation

The linear interpolation, with a first degree polynomial in equation (1), provides a constant speed between two points. If the same time base is used for several motions, the interpolation is linear with respect to each.

For a two dimensional system, like lathe cutting, the path of the tool is a straight line between two points. A taper can be cut this way with an accuracy which depends on the speed regulation and the end point establishment. The E.M.I. (8) cam milling machine, as explained in part II, is provided with linear interpolation between two reference points, whose reference voltages appear at the terminals of the relays r_1 and r_2, as "m" swings from one end of "p" to the other.

An example of linear interpolation in a three dimensional system is the Bendix cam milling machine (3), explained in part II. The variation of the radius between two points is a linear function of the angle between these two points. The tape specifies the magnitude of the angle and the amplitude of variation of the radius between the two points and the machine performs the linear variation according to a process explained in section III, the digital interpolation.

Parabolic Interpolation

The E.M.I. system just referred to above has actually been provided with a "parabolic bridge", that is, the equation (1) is a second degree equation, and the curve thus defined passes by three points: A, B and C (Fig.10) when the tool is running from A to B, then it passes by B, C and D, when the tool is running from B to C, etc. Sharp angles are thus avoided at the points A, B and C etc. However, the curvature of the surface changes abruptly in these points, which is, nevertheless, much better than an abrupt change of direction.

The parabolic interpolation is realized by replacing the potentiometer "p" (Fig.8) by a so-called parabolic bridge (Fig.10). The auto transformer "ab" is equivalent to the potentiometer p previously employed, and picking voltages along this auto transformer would give a linear voltage variation, as AMB on the curve (Fig.10-a).

The regularly spaced taps along the winding "ab" are symmetrically distributed about "mc". The winding mc is the primary of a transformer with secondary windings such as np. The points a, b and c are connected to the relays r_1, r_2 and r_3 providing the voltages A, B and C according to the data from the tape. The voltages NP are distributed along the bridge depending on the number of turns in each winding. If mc has x^n turns, the next winding on each side will have $x^n - 1^n$ turns, the two next ones $x^n - 2^n$ etc. The two last windings have $x^n - x^n = 0$ turns and are a and b themselves. This means that x is the number of secondary windings, np, on each side of mc ($x = 4$ in Fig.10).

The gaps between the different points p are filled by a potentiometer across the sliders of the bridge. The potentiometer is rotated through the medium of the synchronous line "R" (Fig.8) according to the tape feed rate. To have easy changes from one interval between two points to another, two parabolic bridges are used, the control being switched from the first to the second when reaching the point c.

Cubic Analogue Interpolation

The last polynomial interpolation function to be considered will be the third degree equation as developed at the N.A.C.A. Lewis Flight Propulsion Laboratory, for the control of a turbine blade milling machine (10).

If x_{k-1}, x_k, x_{k+1} are the abcissae of consecutive points on the surface of the cam where the radii are, f_{k-1}, f_k, f_{k+1} etc... one can set up the table of Fig.11, where

$$D'_{k-0.5} = f_k - f_{k-1} \qquad D^m_n = D^{m-1}_{n+0.5} - D^{m-1}_{n-0.5} \qquad (3)$$

The interpolating formula is: Eq.(4)

$$f(x) = f_k + D'_{k+0.5}\, t + \tfrac{1}{2}D''_k(t)(t-1) + 1/6D'''_{k+0.5}t(t-1)(t+1)$$

where t varies from zero to one, for x has been changed into x_k - th, with h as the interpolation interval.

Also, f(x) may be written: Eq. (5)

$$f(x) = f_k + (D'_{k+0.5} - \tfrac{1}{2}D''_k - 1/6D'''_{k+0.5})t + \tfrac{1}{2}D''_k t + 1/6D'''_{k+0.5}t^3$$

and the first second and third derivatives of f(x) are:

$$\frac{df(x)}{dt} = D'_{k+0.5} - \tfrac{1}{2}D''_k - 1/6D'''_{k+0.5} + D''_k t + \tfrac{1}{2}D'''_{k+0.5}t^2$$

$$\frac{d2f(x)}{dt^2} = D''_k + D'''_{k+0.5}t$$

$$\frac{d^3f(x)}{dt^3} = D'''_{k+0.5}$$

The required polynomial is seen to be realizable after three successive integrations with respect to "t", starting with $D'''_{k+0.5}$, as shown on Fig.11. After the correct initial values have been set up in the three integrators, it can be shown that a simple adjustment of the third and first integrands is sufficient when proceeding from one interval to the next. This adjustment is the addition of D'''_k to the first integrand and $- 1/6D''''_k$ to the third one. Then "t" has to start over at zero, as necessary for each interval.

In practice, the D''''_k 's, computed from an electronic computer and fed in the machine, are only approximate values, and the simple adjustment described above would involve cumulative errors. Therefore, it has been found necessary to supply the actual data points in order to check the computed values, at the end of the intervals. Any discrepancy detected is corrected during the next interval by adjustment of the third integrand.

Spline Interpolator

This device is the basis of another N.A.C.A. interpolator (10; see also Fig.12). The interpolation curve is given by a flexible metal strip, or spline, constrained to pass through four equally spaced points made by four knife edges. Four servo motors and lead screws position the knives according to the three decimal digit numbers read from punched cards as the co-ordinates of these points. The numbers are converted to voltages by means of precision voltage dividers and the servo motors are driven until the feedback voltages from associated potentiometers balance these data voltages.

The only interval employed is between the two center points. The spline is followed by a probe, through an electric arc whose length is maintained constant by checking and keeping constant the electrical resistance across the gap. The probe follows the spline from point "2" to point "3", then, for continuity purposes, the probe is shifted rapidly to another spline which has been positioned while the first one was traced. The probe then will move from "3" to "2'", and so forth.

Digital Interpolation

The classical example is the numerically controlled milling machine developed by The Massachusetts Institute of Technology (5,10). The heart of the system is the pulse distributor. It produces a pulse train for each of the motions to be controlled. The number of pulses in each train is proportional to the motion required by the instructions given from the tape.

Each pulse in the M.I.T. system calls for an incremental motion of 0.5 thousandth of an inch. These pulse trains are decoded through the use of counting type servos, as described in Section II, to position synchros which supply analogue command information to the actual machine drive servo.

The pulse distributor consists basically of a ten stage binary counter (Fig.13). Each stage in the counter produces two groups of pulses, called "carry" and "non-carry" pulses. The non-carry pulse sets the flip-flop element to one, for it was set to zero earlier. The carry pulse occurs when the flip-flop passes from state one to state zero. It is passed along to trigger the next stage in the counter.

The interspersed "non-carry" pulses, representing transition from zero to one states, are segregated as potential command pulses, and are fed to a row of gates. At each gate is a pulse train with one half the frequency of that on the preceding gate, and no two pulses at these gates are coincident in time.

Whenever the last stage produces one pulse, the first stage produces $2^{10} = 1024$ pulses and any number of pulses between one and 1023 can be selected through the use of the gates. One must first convert the number desired into its binary representation, and the gates corresponding to the 1's in the binary number must be kept open.

The rate of output pulses will be nearly uniform over the entire interval (two seconds in the M.I.T.system) where the 1024 pulses are produced. Actually, in the M.I.T. system, the time interval of two seconds may be changed by the addition of seven additional stages, ad libitum, with then $2^{17} - 1 = 131{,}071$ pulses available, corresponding to a total distance of 65.5355 inches. This makes possible the variations of the feed rate, for a given number of pulses may be obtained in a variable time according to the number of flip-flop sets interposed as specified by the data tape.

As an example, with ten flip-flops one may have 32 pulses in two seconds from the fifth gate starting from the distributor. With 11 gates, the same number of pulses will be produced by the sixth gate, starting from the distributor, but in four seconds. The same distance will then be covered at half speed.

In order to obtain a continuous machine operation, a second set of registers is prepared with information read from the tape while the first set is actively controlling the machine. The end-carry from the distributor, the 1024th one for instance, shifts the control from the first register to the newly filled one.

The independent variable in the M.I.T. system is the time, for the pulse generator always produces 1024 per second. X, Y and Z motions are functions of the time through the use of different numbers of flip flop sets interposed and the specific gates kept "open".

In the Bendix system, described in Part II, the independent motion is the spindle rotation (Fig.5). The axial motion is geared to the spindle rotation and is proportional to the rotation. The cross-slide motion and the spindle speed (in fact, the angle along the helix described by the tool on the surface of the cam) are the two motions to be controlled by the tape (Fig.14).

The pulse generator is then used as a quantizer coupled to the work-spindle, rather than as an oscillator. The output pulses act as command pulses for the servo varying the radial position of the tool in terms of angular intervals, instead of time intervals as it is in the M.I.T. system.

The pulse generator is geared to give $2^8 = 256$ pulses for each five degrees of cam rotation and the eight-stage multiplier produces one end-carry pulse every five degrees. Reducing the number of stages in the multiplier to k would give an end carry pulse every $5 \times 2^{k-8}$ degrees of cam rotation. This is specified by the data from the punched tape. The speed of the spindle rotation is specified directly from the punched data as explained in Part II.

SECTION IV

PUNCHED TAPE AND PUNCHED CARDS

The punched tape or the punched cards represent the only input to be fed into the machine, and must be prepared according to the code of the machine control. Here are some examples to suggest what the problems involved are.

For a stepping device, each hole in the tape means a step forward or backward, according to the hole pattern in the tape. The number of holes on a tape width corresponds to the number of phases of the stepping device, and the distance between holes measured along the tape motion direction is smaller when the desired speed of the motor is higher (Fig.1-C).

Preparing the tape from a table of values is subsequently a straightforward procedure. The inconvenience is that tapes for stepping devices may be very long, as it is the case for the Herman Cousins system explained in Section I. With steps of 0.0001 inch, and signals one eighth of an inch from each other on the tape, a tape over one mile long is necessary for a simple cut of sixty inches. At a cutting speed of one inch per second, the speed of the tape would be over sixty miles per hour.

When closed loops are involved, with feedback, the preparation of the tape is more complicated and one may have to use a digital computer.

For instance, when preparing a tape for the Bendix cam milling machine, radius variations and polar increments have to be computed for points along the helical path H (Fig.4). Points are usually specified in parallel planes spaced widely compared to the pitch of the helix.

An interpolation and co-ordinate transformation therefore is required to determine points along the cutting path. Once these points are known, the difference must be taken to give the "Dr", multiplied by a constant to convert to machine increments, and expressed in binary notation. For security purposes a parity check must also be determined to make the total number of holes in the two-line block of data even (Fig.15-a). Standard punched office accounting machines may be used for these operations; the input of the computing machine is manually punched cards for each initially given point. The output of the computer is a card for each point of the tool path, containing all the informations in exactly the form required for the control tape. The tape is punched automatically from the cards by special equipment.

The entire process requires a few days for preparation of the tape and one day is usually the maximum time required for cutting the first cam.

The configuration of the final tape is as on (Fig.15-a). Special instructions may be given to the machine to run the tape n times, with progressive advance of the tool, to limit the depth of each cutting pass.

The control tape of the M.I.T. milling machine has only seven hole positions across its width. The instructions for a point are given in a block (Fig.15-b). The first row in a block specifies the length of the pulse distributor and consequently the time interval. The next row gives the direction of motion for Dx, Dy and Dz. These quantities are arranged longitudinally, on the tape, least significant digit first, each in a separate channel, with ten to seventeen digits, according to the length of the pulse distributor. Four channels remain free besides the three used for Dx, Dy and Dz and are used for checking and self correction purposes. A single error can be detected and corrected in each row of hole positions, a double error can be detected, and the machine then stopped.

To modify a program, another tape must be prepared, and run every time a piece has to be cut. The changes or alterations in a piece are more easily performed when the program is read from cards than from tape. This can be explained with the example of the General Electric punching machine previously quoted in Section II (9).

The input cards are 45 column tabulating cards of the Remington Rand type (Fig.15-c). Each card specifies one machine operation. All instructions are given in decimal digital form and are expressed in hundredths of an inch with respect to zero reference. For instance, the number 01500 indicates 15,000 inches.

A five digit number is also used to express the angular position of the turret, with respect to zero reference. A complete rotation is divided in 10,000 parts. Thus, for instance, 5,000 indicates a rotation of 180 degrees.

The tabulating card contains also the following information: part or drawing number, sequence number of the card and some machine and operation instructions. The cards are read in a Remington Rand card reader to which a push button control station has been added. The reading pins pass through the holes of the card then are locked, and an extra push of two-tenths of an inch with a force of two pounds is used to operate electrical switches. The readings are then transferred into position detector signals for the three servo mechanisms as explained in Section II.

The column for machine instruction provides for the orders of positioning and punching, positioning without punching, center punching (to mark operation to be performed later in a definite point) etc. As all the cards contain a sequence number, the machine stops when a card happens to be out of sequence, unless a special order has been given to prevent the counter from advancing as it should do, thus permitting the introduction of cards modifying the program established earlier. The sequence counter may be reset to zero and advanced next to any number, to permit the machine operator to start the sequence at any point.

The column devoted to operator's instructions is used whenever the work clamps have to be moved, or when the position of the work sheet has to be changed. A signal light appears on the control board to transmit the indication to the operator.

If a group of pieces has to be worked with slight differences between them, the same deck of cards can be used for all parts of the work which is common to them, and for the other parts, a group selection control will accept only the type of cards specified by the switching board as instruction cards, while the other cards are passed through the machine at maximum feed rate.

SECTION V

COMPARISONS, INTERESTS, ADVANTAGES

The various systems of digital-to-analogue conversion explained above meet different industrial needs.

The stepping devices, for instance, are the simplest of all as far as the principle of operation is concerned and it seems that the first numerical control devices were of that type. It is also possible to handle problems of high complexity: it was seen that the Herman Cousins system can handle four independent variables at one time.

These systems, however, have several inconveniences. First, a slow speed in machine member motions, together with a correlative high speed of the tape control for an acceptable speed of the machine motion, next, the interpolation between two consecutive points is performed only by the inertia of the machine members in motion at that time. The number of data points must therefore be rather high if some accuracy is required, with the consequence of a lower speed.

Third, as explained in Section II, the power output of a stepping device is generally low, unless a servo mechanism is provided to help the stepping motor to drive the part to be moved. If such is the case, the servo mechanism together with the concerns of a feedback loop with stability and speed of response problems make the solution of the same complexity as any other numerically controlled device. Therefore, stepping devices are mostly used for continuous control of slowly varying motions rather than for end-point controls, where shifts must be made rapidly from one position to another.

Using a quantizer implies necessarily a measuring device (pulse generator, optical gratings, etc.) and servo mechanisms, with feedback loops. Strict requirements on overshoot and speed of response add to the complexity of the problem. The same requirements hold for coded digitizers and matrix decoding devices.

The principal disadvantage of systems using quantizers is that an accurate knowledge of position depends on having correctly counted all increments from the last known reference point. However, additional means to avoid this kind of difficulty is frequently justified by the relative simplicity of quantizers as compared with absolute digitizers.

It has been shown in Part II that some difficulty was encountered when "reading" the indications from the digitizer, for one contact brush may straddle more than two bars on the disc (Fig.7-a). A sort of balance exists therefore between quantizer and digitizer systems, with their respective inconveniences and advantages.

A compromise may be reached between these two devices by the use of an input in coded form to preset a counter, and by controlling the end-points with a digital quantizer which pulses the counter down to zero as the desired location is being reached.

The best accuracy, however, seems to be obtainable from matrix decoding systems, with the use of multi-speed data transmission systems, as explained in Section II. The accuracy of the control system is proportional to the number of pairs of selsyns employed. The maximum in that way has been performed for military purposes; applications for radar has been realized with seven pairs of selsyns, and perhaps better, more recently.

The large variety of systems with their own inconveniences and advantages can be explained only by the variety of the problems to be solved. End-point location is easily solved by coded digitizer systems, while continuous control systems using quantizers or matrix decoding meet the requirements for continuous control, and stepping devices are interesting for small output powers in continuous control and machining of great complexity. In any specific case, economical considerations may modify the choice of one of these various machines for a given job, but this consideration is beyond the scope of this study.

All the systems described above tend to the solution of the same problem: to eliminate as much as possible the human error factor from machining.

A human operator can only repeat a given sequence of operations so many times without an error, and the intricacy of sequencing, frequently indicating an expensive and complicated job, is directly related to the frequency of the error. Sometimes the intricacy of a piece makes it too difficult for a man to work it without error, but a special set up of an automatic machine is not economical for the number of pieces involved. However, a single prototype could be made by skilled craftsmen, using a high accuracy machine far below its possibilities of production with respect to feed rates, cutting speeds and so on. A large part of the time would be spent in reading the blue-print, with possibilities of misunderstandings and errors amidst the distracting noises of all sorts in the shop.

Numerically controlled machines have brought an easy solution to all these problems. Whatever the complexity of a piece may be, programming a tape presents quite the same difficulty, which is never tremendous, and is always performed in the office away from any distracting noise. The procedure could be practical even for one piece. The time lost in reading the blue-print no longer exists, safety itself is improved for the operator is freed from putting his hands into the work area, and the hood, eventually closed and open from the orders given by the tape (or the cards), is fully efficient.

Setting up an automatic machine heretofore required a skilled operator, or a "set-up" man. Now the set-up is planned by a method engineer or programmer and the tape or cards are prepared by a clerk, both in the office. The tape is delivered to the machine operator with the shop order and operation sheet or tool scheme. An unskilled operator can mount the numbered "plug-in" preset tools according to the chart, place the tape in the machine director, and the machine is ready to run.

After completion of the run, the tape can be stored in a filing cabinet drawer for future use such as a new order to replace broken parts or for spare parts, thus saving the trouble of costly storage. Mailing the tape could even replace the mailing of spare parts.

The improvements in industry due to numerically controlled machine tools could be summarized in the five following points:

Improvement in speed, accuracy and elimination of random errors.

Improvement in preparation, modification, reproduction and storage of instructions, whatever the complexity of the problem may be.

Improvement in set up time between jobs and efficiency of skilled workmen.

Improvement on machine productivity and manufacturing cost

Extension of the field within which a given machine can be used economically.

APPENDIX

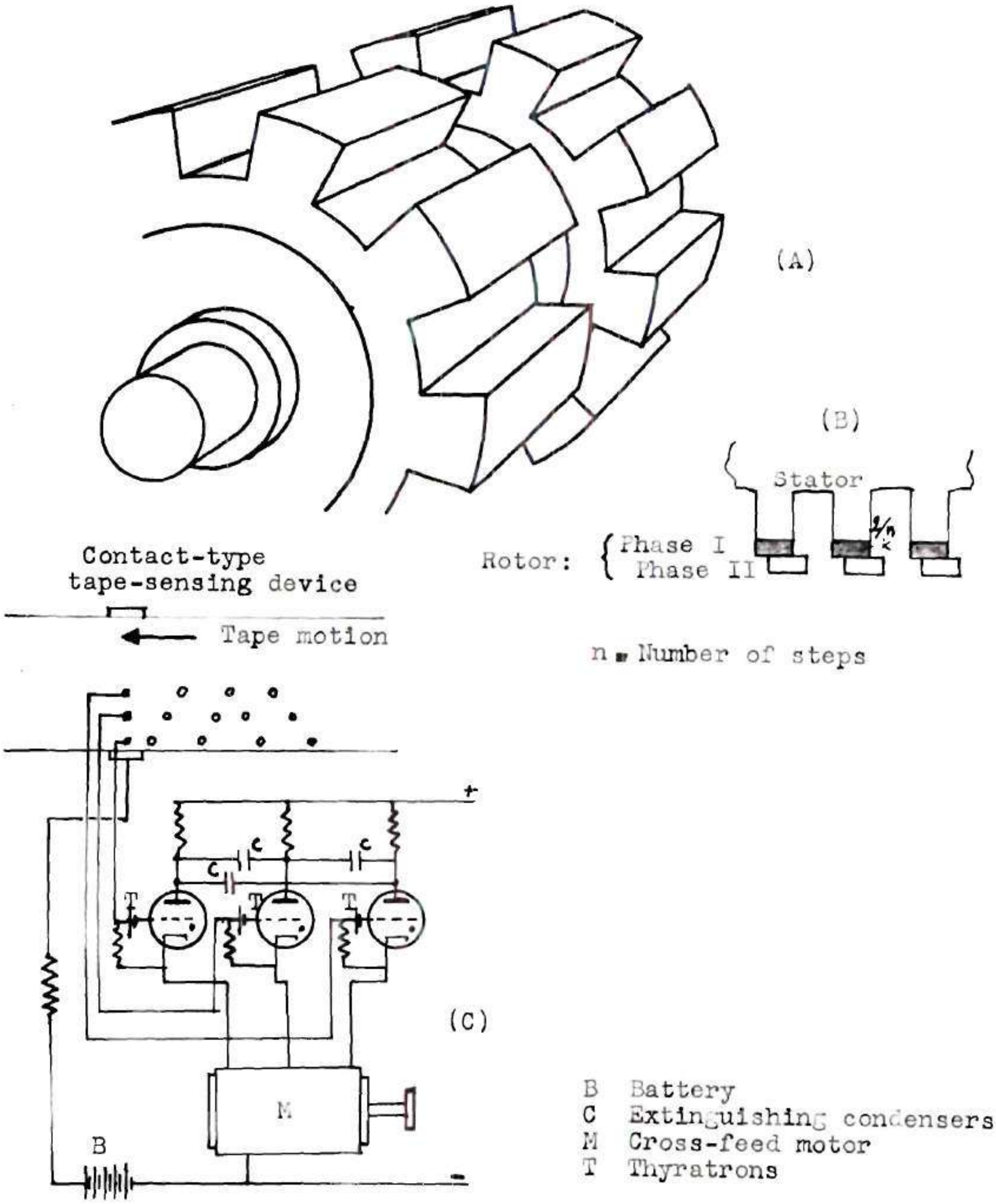
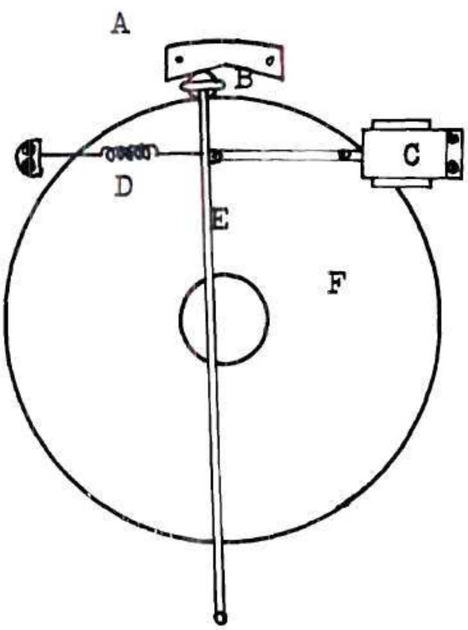


FIGURE 1: (A) ROTOR of the STEPPING MOTOR

(B) POLES CONFIGURATION of the MOTOR

(C) TAPE PROGRAMMING UNIT FOR A STEPPING MOTOR



A Vee-shaped cam
B Roller cage
C Brake reversing solenoid
D Spring
E Pivot arm
F Hardened disc

FIGURE 2 : ONE WAY BRAKE ON STEPPING MOTOR

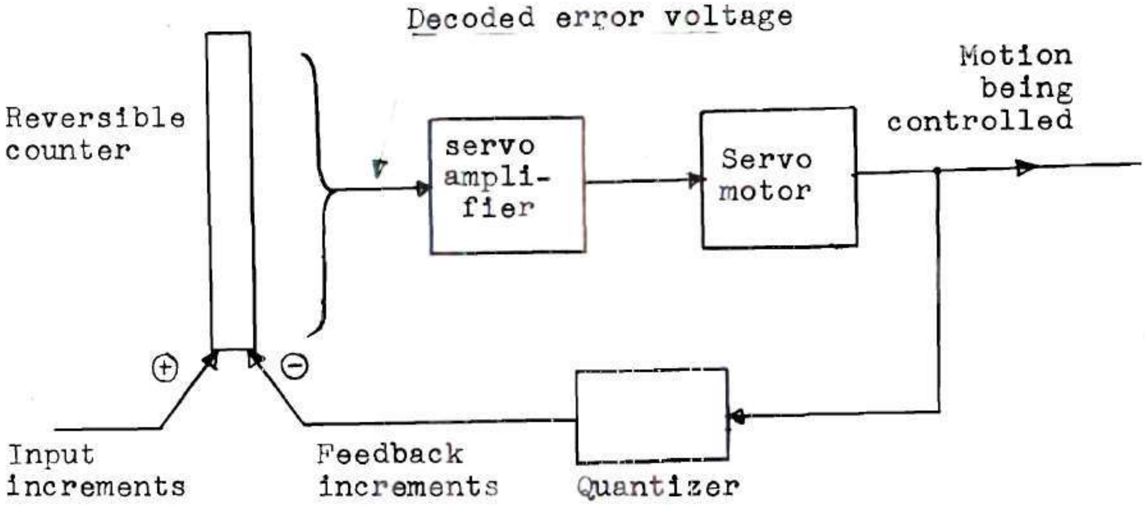


FIGURE 3 :CLOSED-LOOP INCREMENTAL SYSTEM USING A QUANTIZER AS A FEEDBACK INSTRUMENT

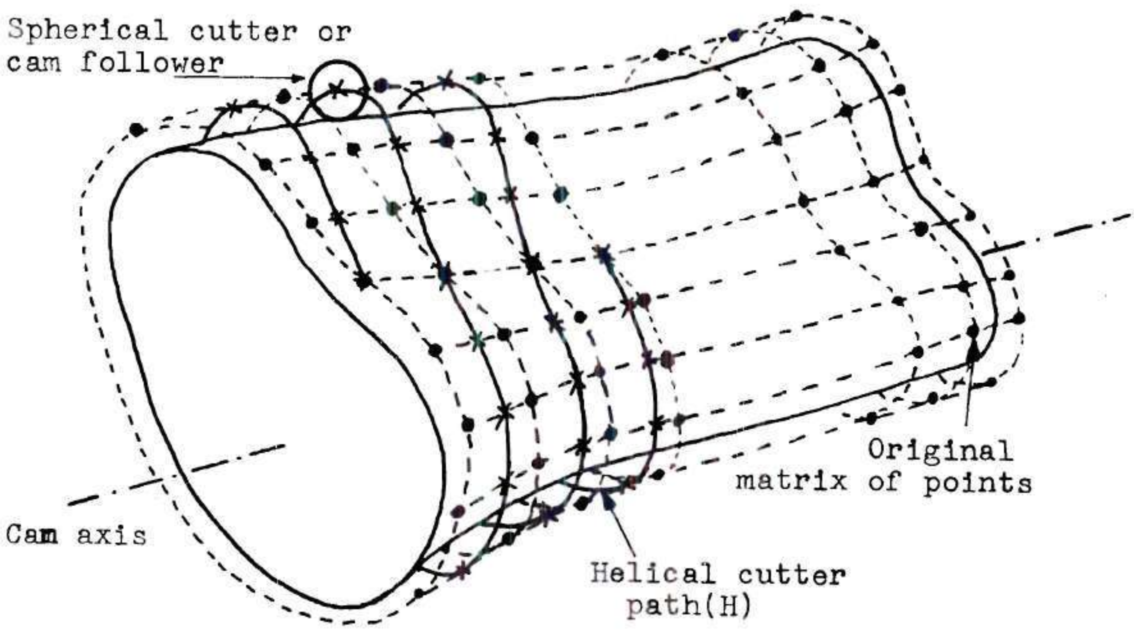
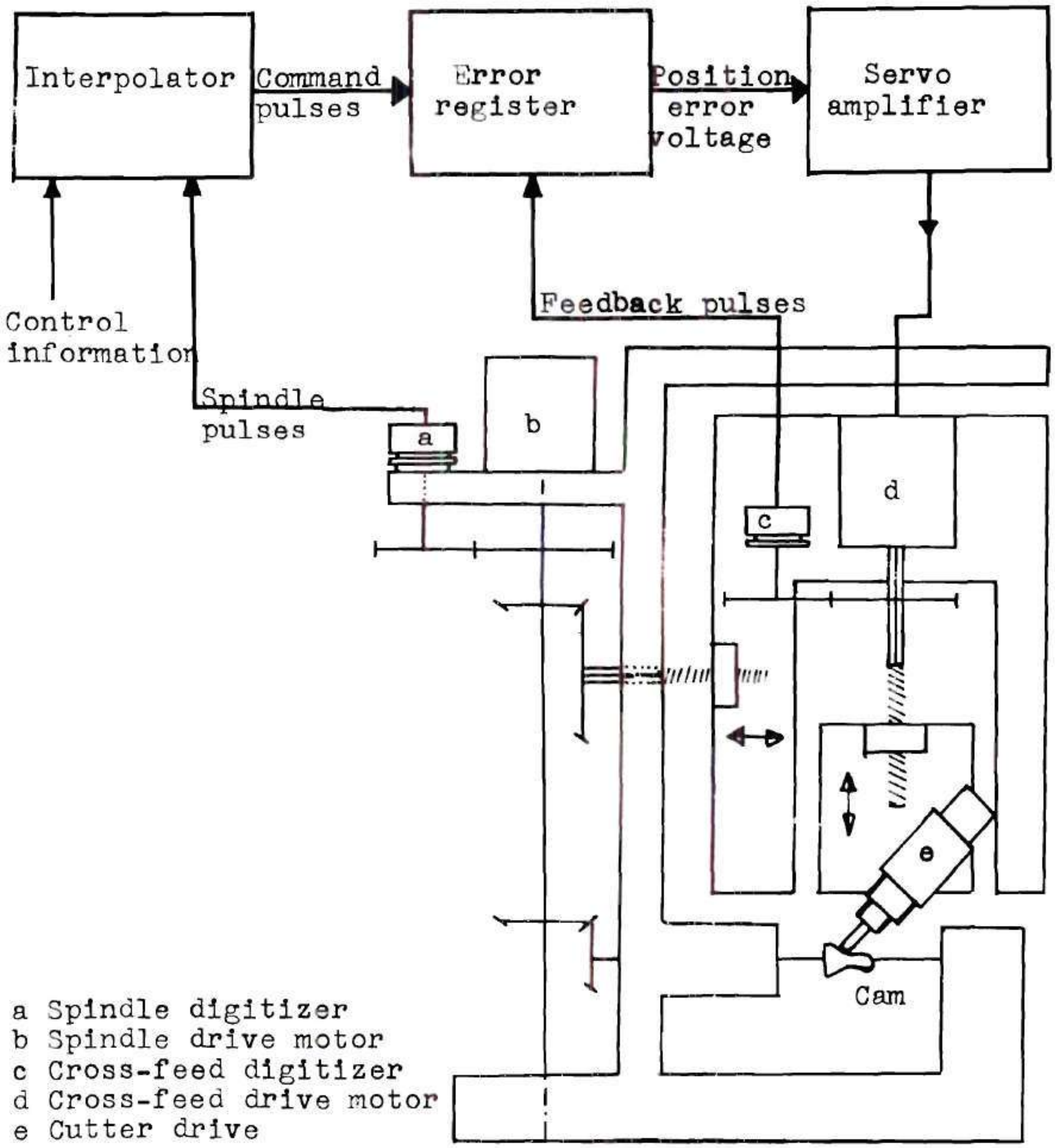
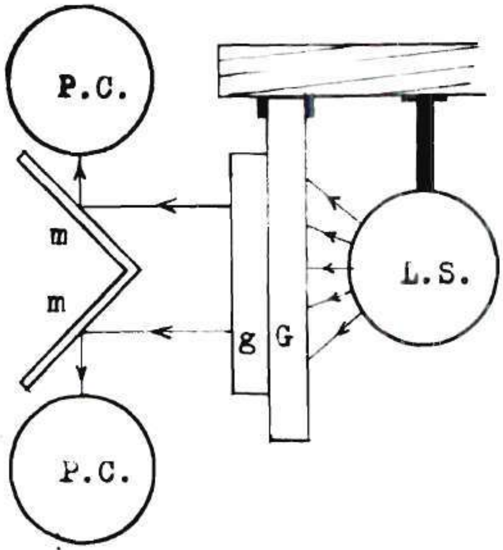


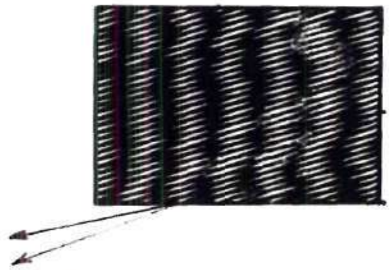
FIGURE 4 : GENERALIZED CAM GEOMETRY



a Spindle digitizer
b Spindle drive motor
c Cross-feed digitizer
d Cross-feed drive motor
e Cutter drive

FIGURE 5: SCHEMATIC DIAGRAM OF THE BENDIX NUMERICALLY CONTROLLED MACHINE SYSTEM





Angle of the gratings giving a moirè pattern

P.C.	Photo cell
L.S.	Light source
m	Mirror
G	Moving grating
g	Fixed grating

FIGURE 6 : OPTICAL GRATING FOR HIGH ACCURACY LINEAR MEASUREMENTS (Cf FERRANTI SYSTEM)

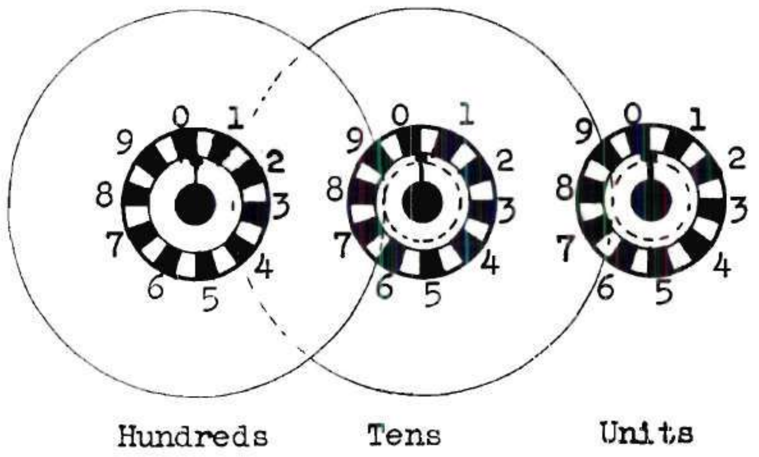
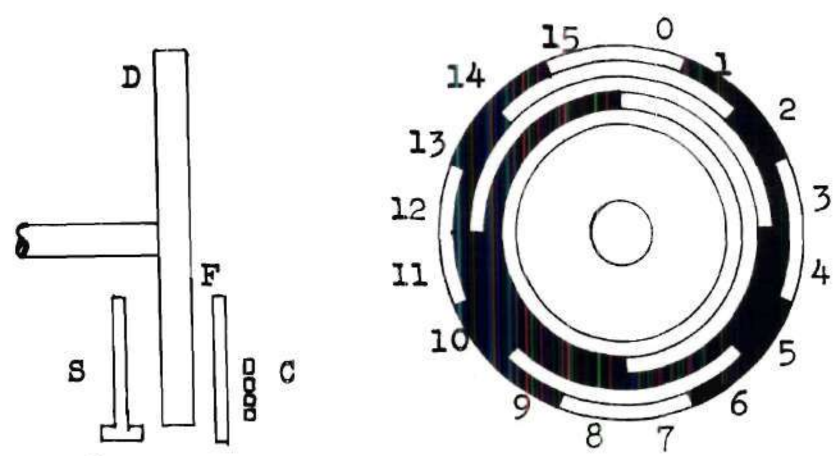


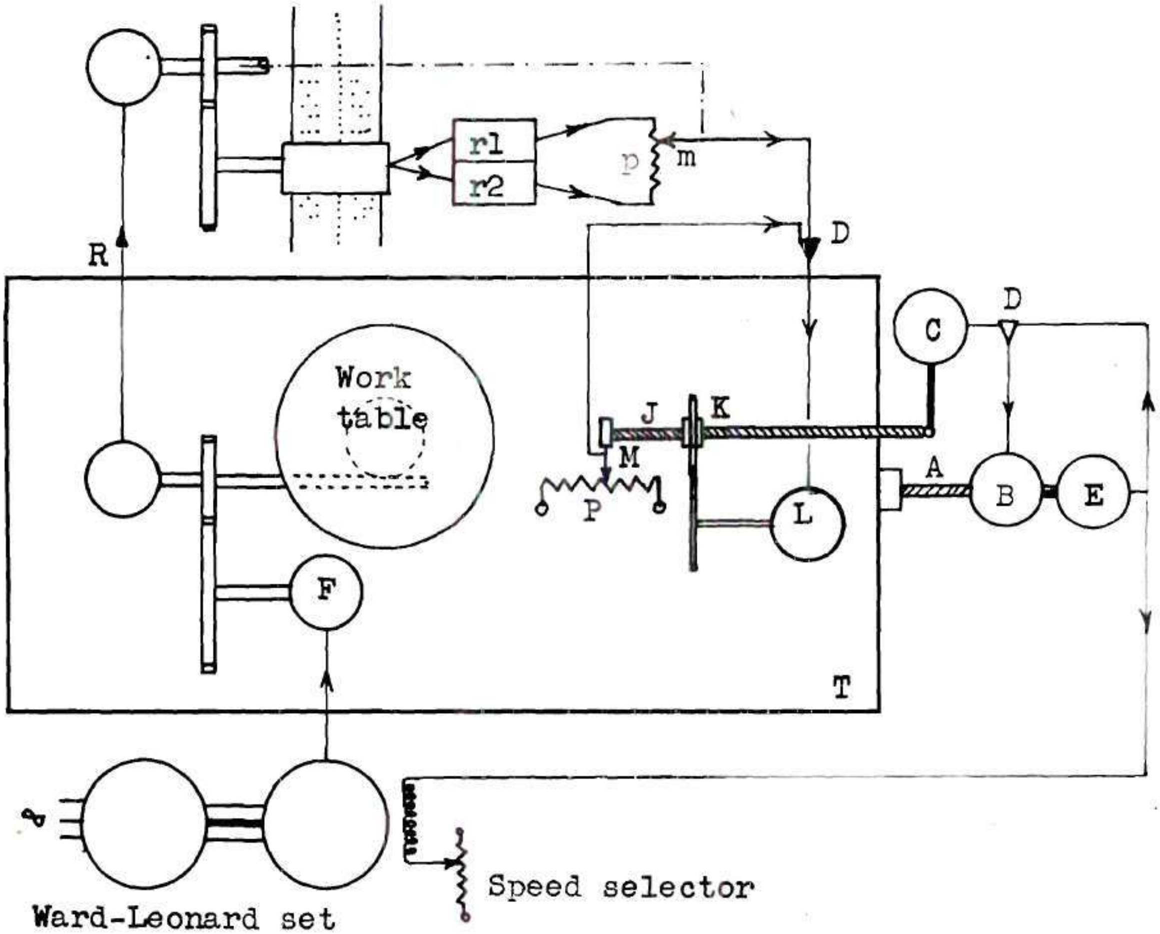
FIGURE 7- a : DIRECT READING FROM CONTACT BRUSHES

DECIMAL COMMUTATORS



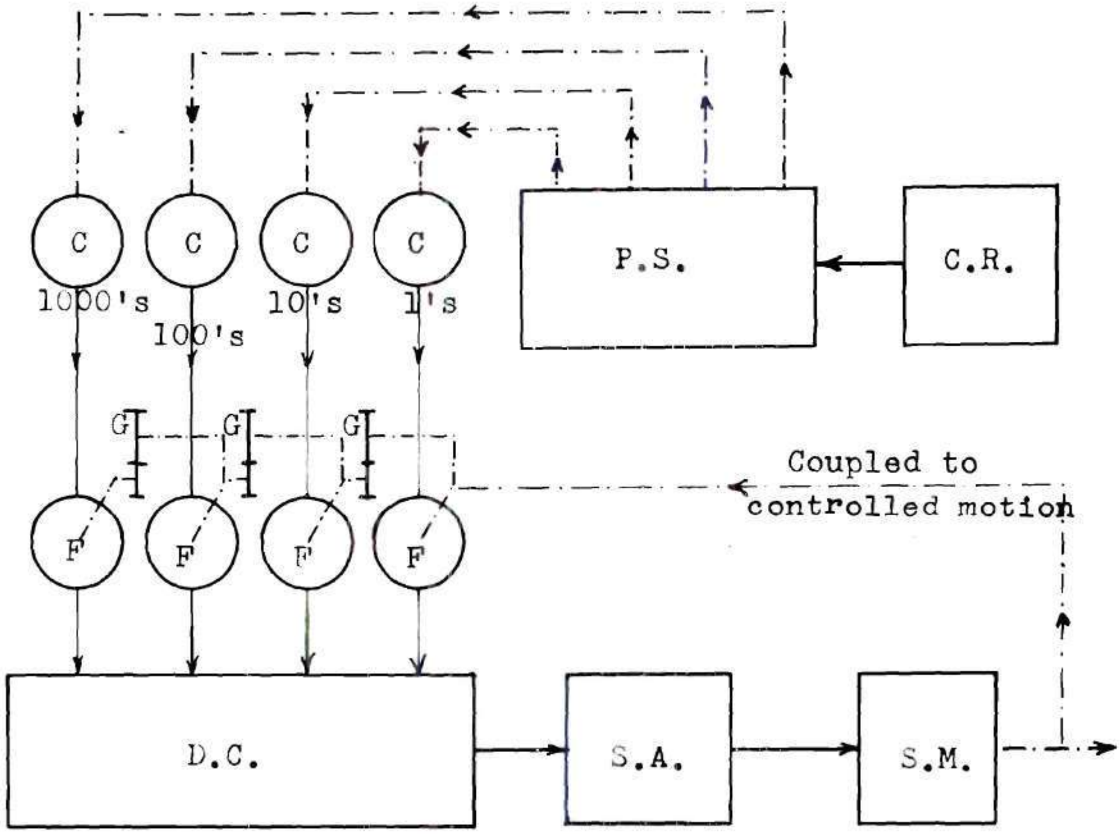
C Photocells
D Disc
F Slit
S Light source

FIGURE 7 - b : DIRECT OPTICAL READING . BINARY CODED DISC



A Leadscrew
B Servo-motor leading A
C Follower actuated by J and leading B
D Amplifiers
E A.C. generator. Acts as a speed limitator
F Motor leading the work-table
J Leadscrew
K Nut fixed on cross-slide T
L Servo-motor leading J
P Potentiometer indicating the position of T
p Potentiometer indicating the desired position.
R Synchronisation between work-table and tape-reader speeds
T Cross-slide

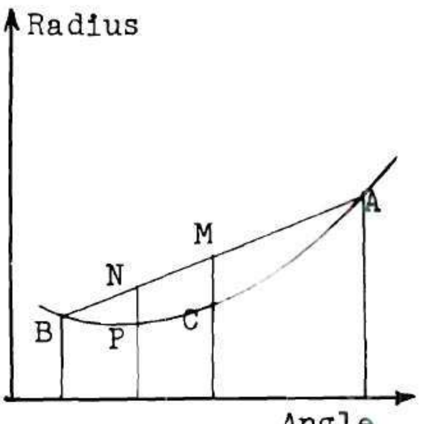
FIGURE 8 ; E.M.I. CAM MILLING MACHINE



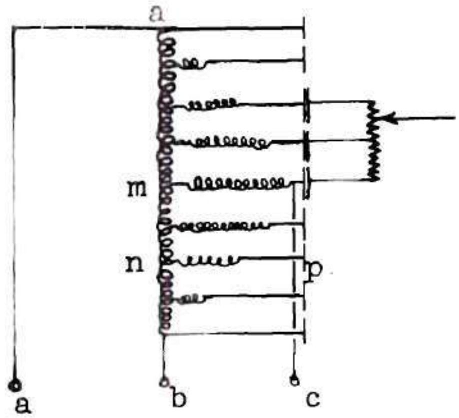
.—.—.—. Mechanical links
———— Electrical connections

C.R. Punched card reader
P.S. Command-synchro positioning system
C. Command synchros
F. Feedback synchros
G. Gear reduction
D.C. Discriminator circuit & switching network
S.A. Servo amplifier
S.M. Servo motor

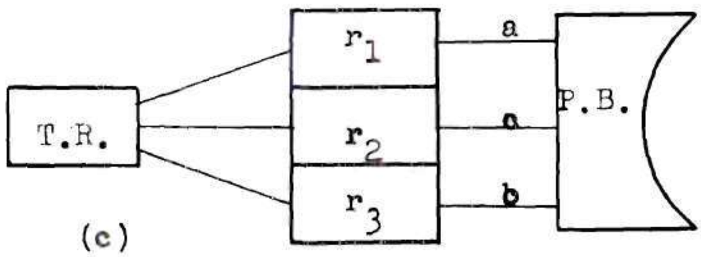
FIGURE 9 : MULTI-SPEED SYNCHRO APPLIED TO A MACHINE-TOOL CONTROL



(a)



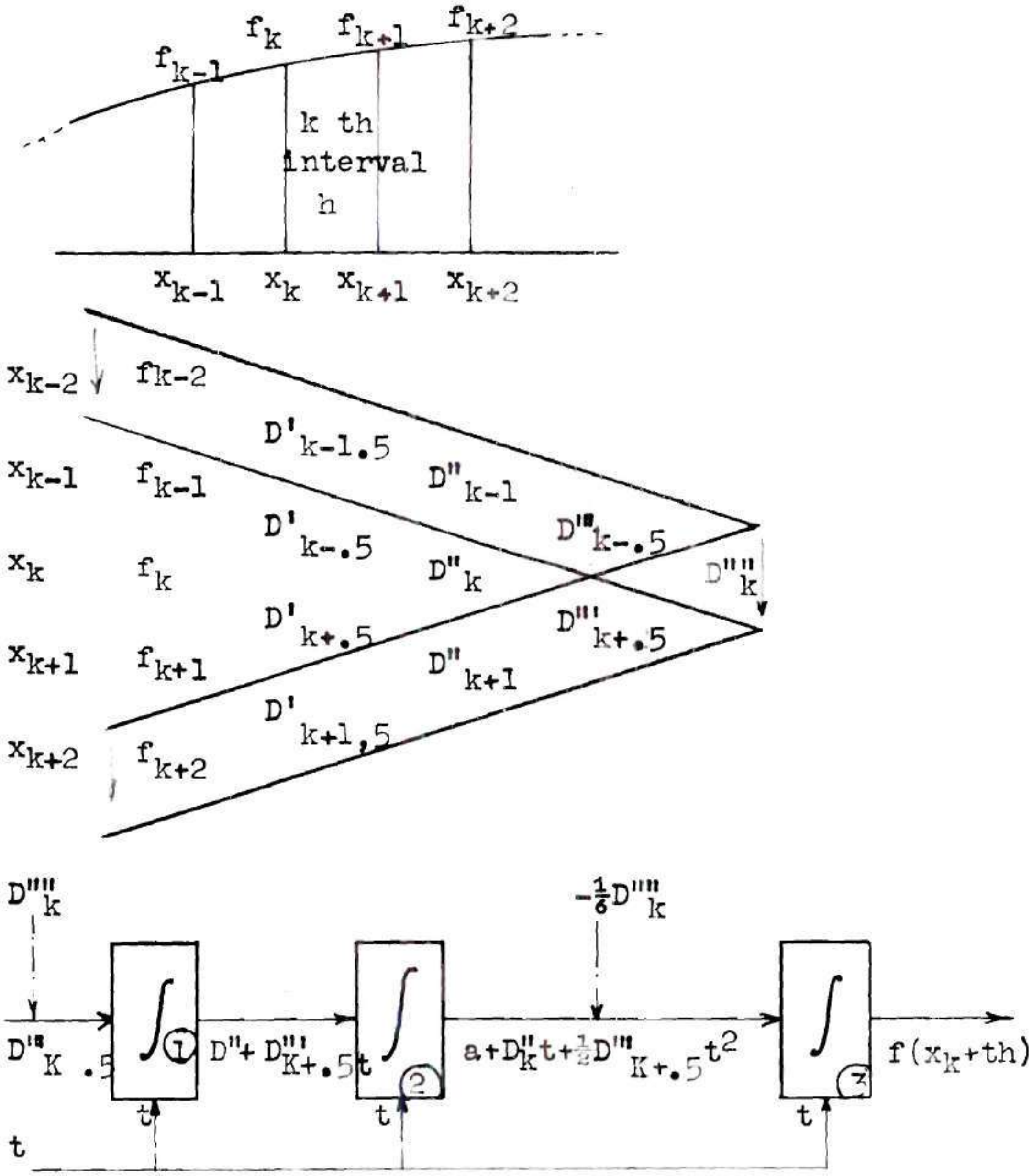
(b)



(c)

(a) Voltage diagram across the parabolic bridge
(b) Parabolic bridge circuit
(c) Connection: Tape reader, Relays, Parabolic bridge
(T.R.) ($r_{1,2,3}$) (P.B.)

FIGURE 10: THE INTERPOLATION SYSTEM OF THE E.M.I. CAM MILLING MACHINE - PARABOLIC BRIDGE

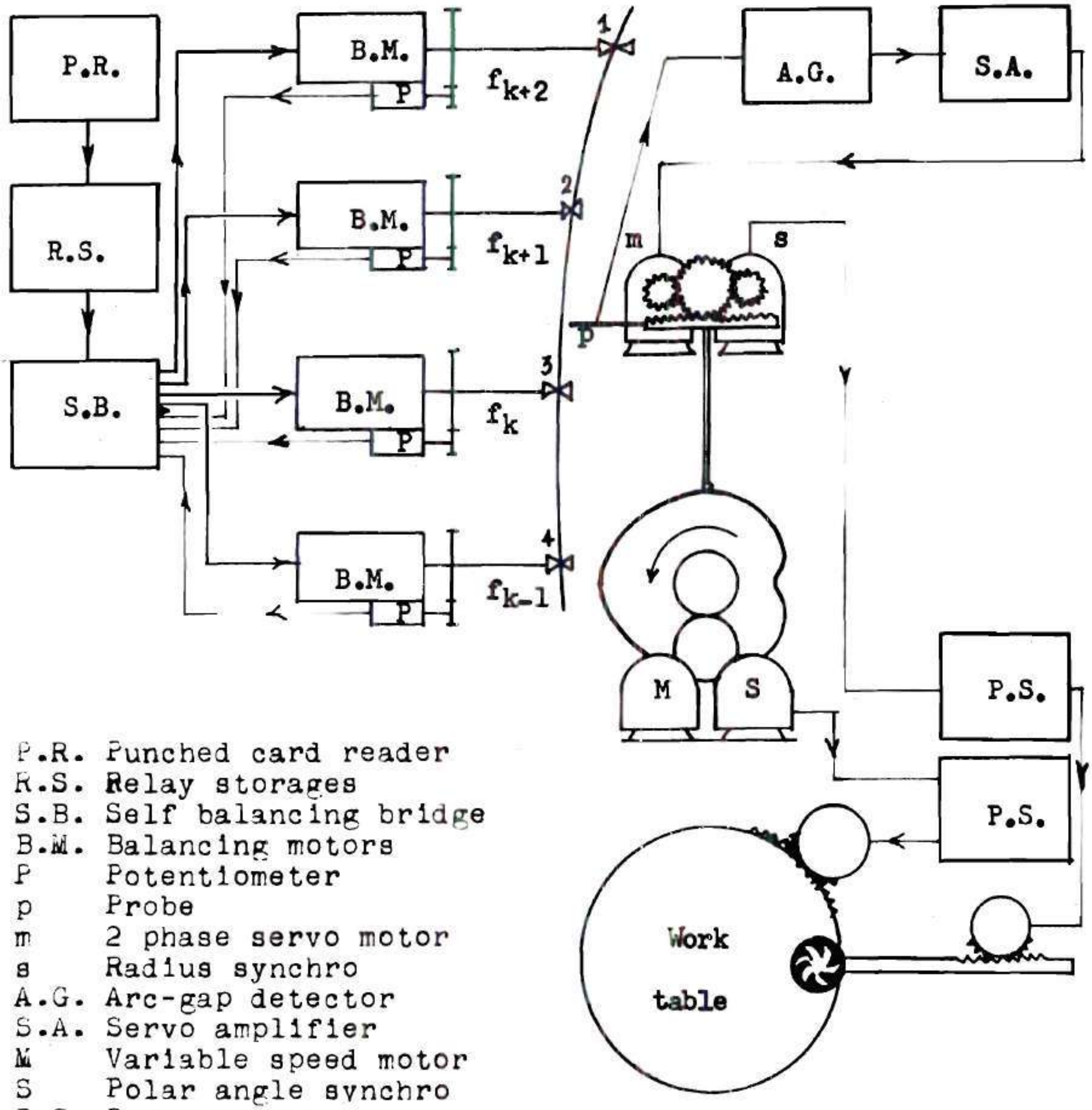


$$a = D'_{k+.5} - \tfrac{1}{2}D''_{k} - \tfrac{1}{6}D'''_{K+.5}$$

$$f(x_k + th) = f_k + at + \tfrac{1}{2}D''_{k}\,t^2 + \tfrac{1}{6}D'''_{k+.5}\,t^3$$

FIGURE 11: THIRD DEGREE INTERPOLATION SYSTEM

WITH THE INTERCONNECTION OF INTEGRATORS



P.R. Punched card reader
R.S. Relay storages
S.B. Self balancing bridge
B.M. Balancing motors
P Potentiometer
p Probe
m 2 phase servo motor
s Radius synchro
A.G. Arc-gap detector
S.A. Servo amplifier
M Variable speed motor
S Polar angle synchro
P.S. Power servo

FIGURE 12 : SPLINE INTERPOLATOR APPLIED TO A MILLING MACHINE

BUILT BY N.A.C.A.

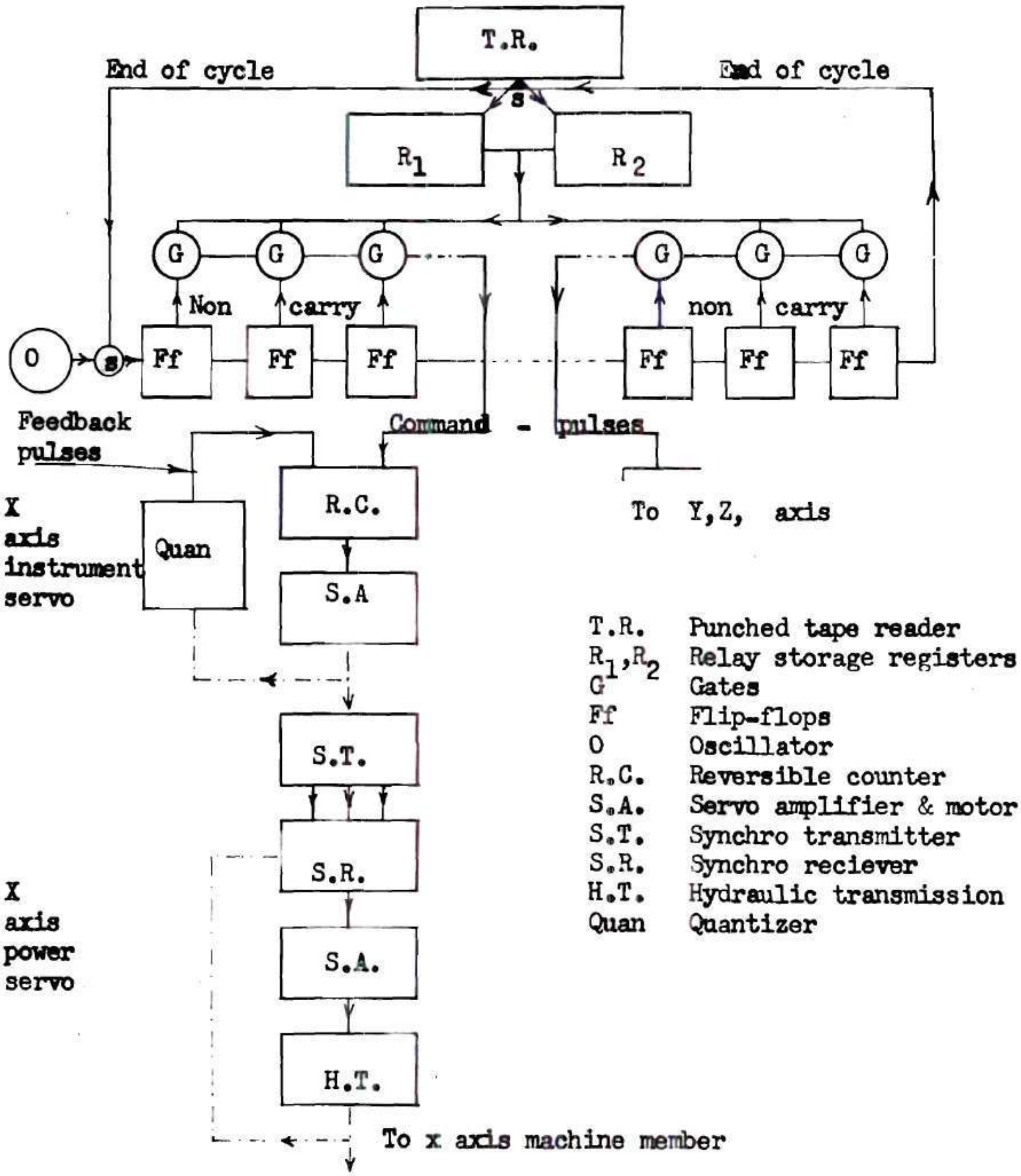
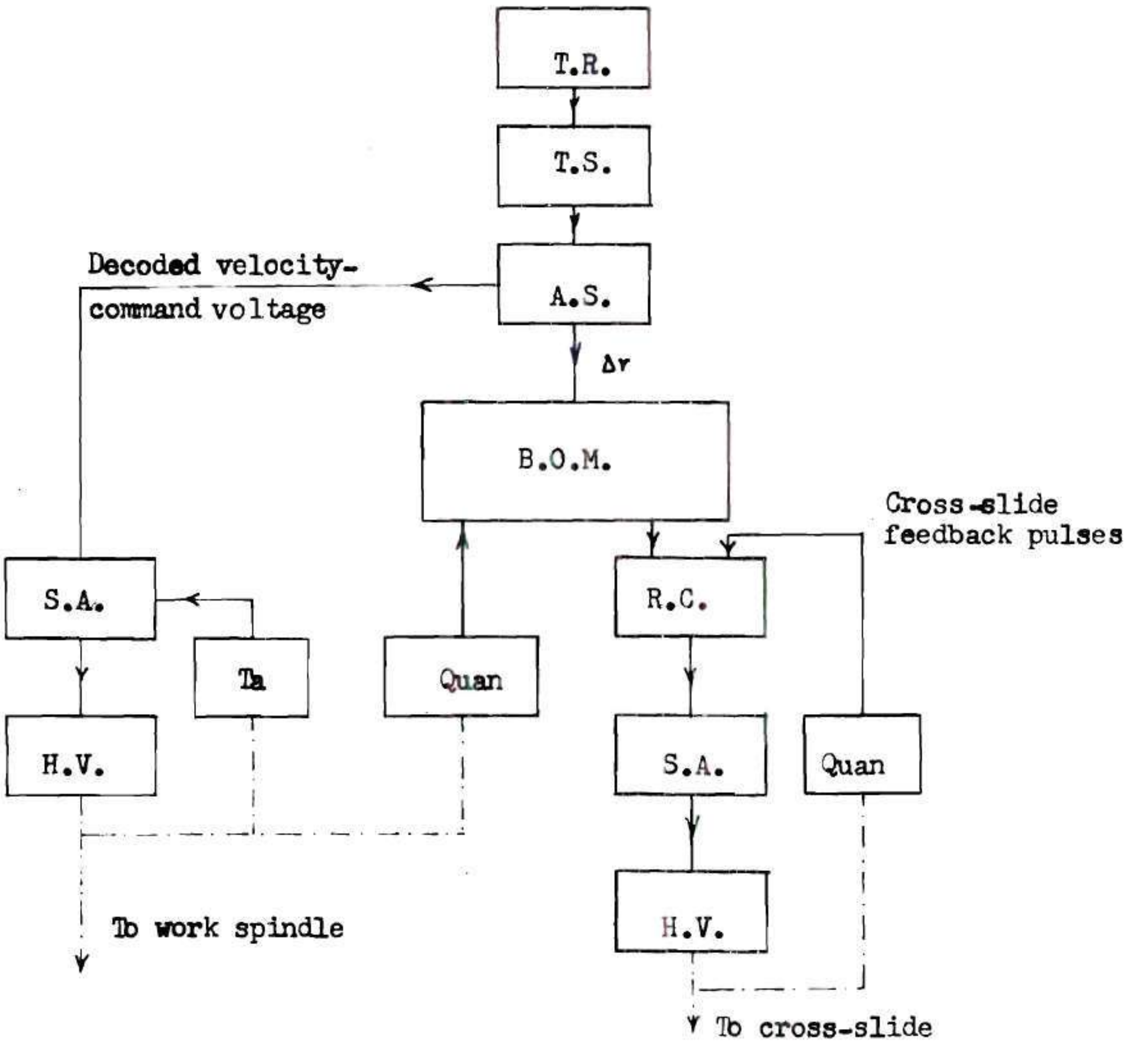


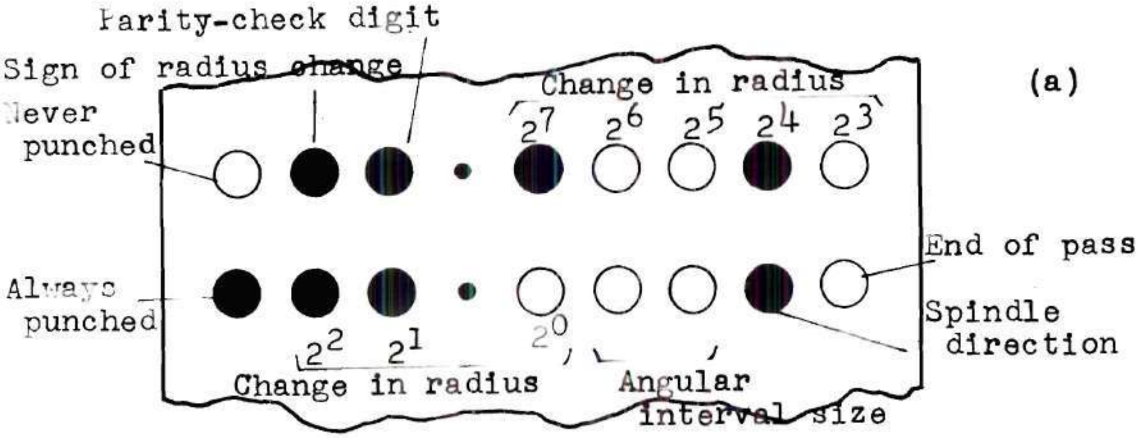
FIGURE 13 : SCHEMATIC DIAGRAM OF THE NUMERICALLY CONTROLLED MILLING MACHINE BUILT AT M.I.T.



T.R. Punched tape reader
T.S. Temporary storage register
A.S. Active storage register
B.O.M. Binary operational multiplier
R.C. Reversible counter
S.A. Servo amplifier
H.V. Hydraulic valve and motor
Quan Quantizer
Ta Tachometer

FIGURE 14 : SCHEMATIC DIAGRAM OF THE NUMERICALLY CONTROLLED CAM-MILLING MACHINE (BENDIX)

(a)



(b)

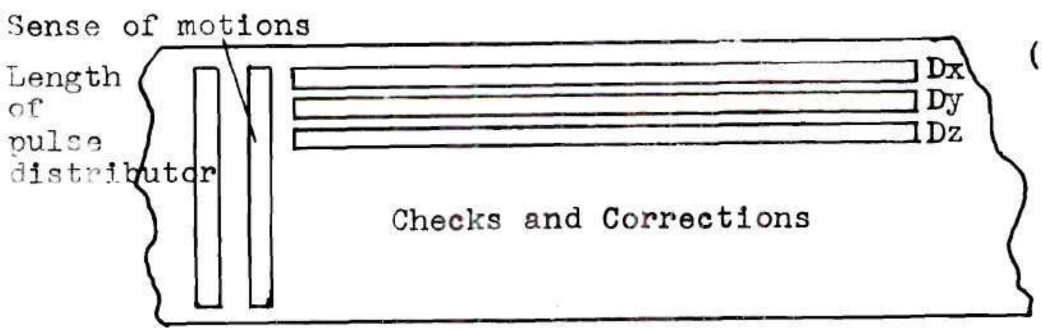
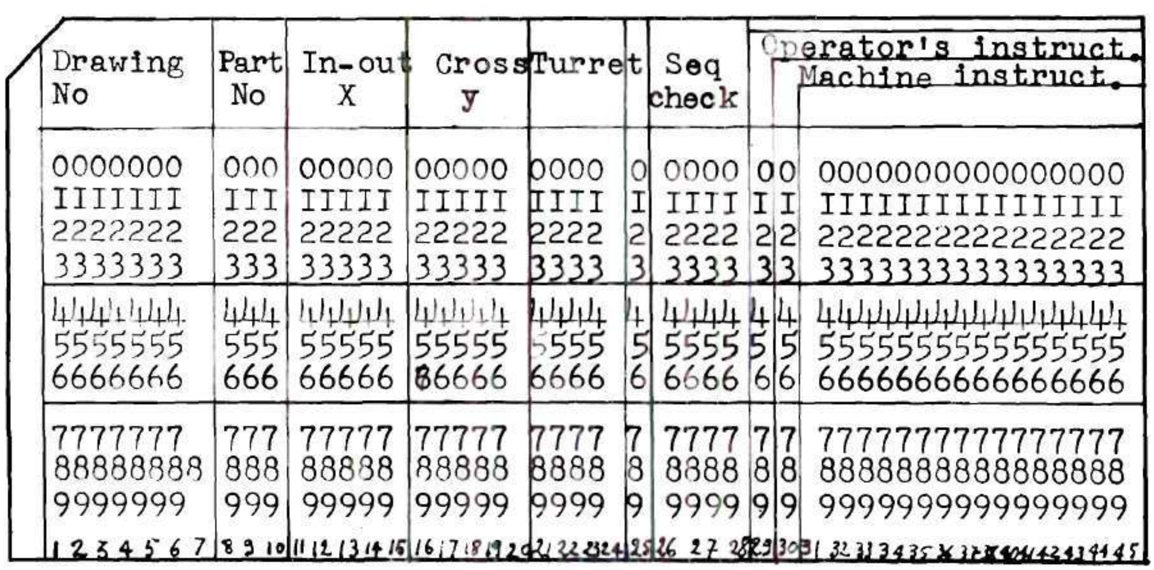


FIGURE 15: PUNCHED TAPES & PUNCHED CARDS

(c)



BIBLIOGRAPHY

(1) F.W.Cunningham, "Controlling Machines With Tape", Automation, Vol. 1 (August, 1954), pp.77-82.

(2) A.G.Thomas and Fred J. Fleishauer, "Power Stepping Motor, a New Digital Actuator", Control Engineering, Vol.4 (January, 1957)

(3) E.C.Johnson, "A Numerically Controlled Cam Milling Machine", Proc..National Industrial Electronics Conf., September, 1955

(4) D.T.N.Williamson, "Computer Controlled Machine-Tool", Engineer, Vol.200 (July 1955), p.151.

(5) E.M.Grabbe, Automation in Business and Industry, New York: John Wiley & Sons, Inc., January, 1957.

(6) "Control in Two Dimensions", Automatic Control, Vol.1,(August, 1954), pp.24-25.

(7) "Boring Unit...Tape Controlled", Automation, Vol. 2, (September, 1955), pp.69-70.

(8) Electric & Musical Industries Ltd., "E.M.I.Development in Computer-Controlled Machine-Tools", Machinery, London, Vol.87, (July 15, 1955), pp.144-150.

(9) L.U.C.Kelling, "Numerical Control of Machine-Tools", Institute of Radio Engineers, Transaction on Industrial Electronics, March, 1955.

(10) E.C.Johnson, Application of Computer Techniques to Machine and Process Control, Detroit, Michigan: Bendix Aviation Corporation, Research Laboratories Division, December, 1955.